International Journal of Mechanical and Production
Engineering Research and Development (IJMPERD)
ISSN(P): 2249-6890; ISSN(E): 2249-8001
Vol. 8, Issue 6, Dec 2018, 773-782

THERMAL ANALYSIS OF VISCOELASTIC PROPELLANT GRAINS WITH DEVELOPED AXISYMMETRIC FINITE ELEMENTS USING HERRMANN FORMULATION

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ABSTRACT

Solid Rocket Motor (SRM) is developed based on casting method where solid propellant grains are cast into a composite or metallic casing. Generally, SRMs are exposed to extreme loading scenarios during storage, transportation, and firing, leading to cracks in the solid propellants. In this paper, Computational Finite Element Analysis is performed with developed 8 node quadrilateral, 9 node quadrilateral and 6 node triangular elements using Herrmann formulation to analyze stress and strain variations in the head and mid segments of the SRM subjected to thermal loading. The obtained results are compared with commercially available Finite element software.

KEYWORDS Solid Rocket Motor, Herrmann Formulation & Thermal Loads

Received: Oct 09, 2018; **Accepted:** Oct 29, 2018; **Published:** Dec 14, 2018; **Paper Id.:** IJMPERDDEC201878

Original Article

INTRODUCTION

The structural design of Solid Rocket Motors (SRM) is conceptually based on weak solid propellant grains cast inside a strong composite or mechanical case. The case provides structural resistance against loads and the low strength propellant grain inside is used for transmitting loads for the surface of the grains to the case. Generally, SRM is exposed to diverse loads during its life. It is a known fact that due to these loading conditions cracks can develop. Hence, to determine the structural integrity and service life of the SRM, studies were performed to evaluate the stress and strain distribution [1]. Computational Finite element methods have the capability to solve complex loading conditions, material properties, and complicated geometries. The finite elements are classified as Displacement Method [2], Force Method [3], and Mixed Method [4]. The major structural components of the rocket motor are motor case, incompressible liner, and propellant. The study of the viscoelastic structure can be reduced to an elastic analysis by following the concept of Schapery [5] described in detail by Kanakaraju et al. [6].

The finite element developed by displacement method was studied by many researchers [2]. The disadvantages of this method are that it cannot handle the nearly incompressible materials like that of the solid propellants. For such materials (Poisson Ratio approximately 0.5), the strain components are not independent anymore. Therefore, the principle of minimum potential energy and the corresponding displacement method experience locking and the resulting solutions and erroneous [7]. Locking occurs when the element formulation is not sufficient for capturing the appropriate displacements. In these materials, the volumetric strain is nearly zero, hence using displacement method based finite elements results in zero displacement and the calculated stresses are

under predicted and unreliable when low order displacement interpolations are used. Although higher order interpolations, such as biquadratic interpolation or mesh refinement may be used, the displacement solution is generally not accurate. The solution to element locking in case of incompressibility is to break the strain field down to its fundamental components. In the case of any deformation, there are deviatoric and volumetric strain components. Deviatoric strains determine the shape change of the body and volumetric strains determine the volume change (dilatation) of the body. The volume change occurs due to a hydrostatic pressure. The trouble caused by the displacement based finite element formulation for the incompressible material can be understood by examining the familiar elasticity relationship,

$$\frac{Bulk\ \mathrm{mod}\,ulus}{Shear\ \mathrm{mod}\,ulus}=\frac{K}{G}=\frac{2(1+\nu)}{3(1-2\nu)} \tag{1}$$

Where $K=\frac{E}{3(1-2\nu)}$ and $G=\frac{E}{2(1+\nu)}$, E is Young's Modulus and ν is the Poisson's Ratio. For nearly incompressible materials, the Bulk modulus becomes large relative to the Shear modulus. In the limit, when the material is completely incompressible (ν=0.5), all hydrostatic deformations are precluded. In this limiting case, it is therefore not possible to determine the complete state of stress from strain alone. Therefore, special formulations are required to account for the hydrostatic deformations as well as to predict the actual state of stress for such materials. Thus, in this study, a computational study based on Herrmann Formulation is presented to overcome the limitation of the direct method. Here an 8 node quadrilateral, 9 node quadrilateral, and a 6 node triangular element are developed and studied for stress and strain variations for head and mod section of SRM subjected to the thermal load.

FORMULATION

In this paper, the Herrmann formulation code is written using visual C++ language and validated with analytical solutions available in the literature. The elements developed are 8-node and 9-node quadrilaterals and 6-node triangular elements having linear pressure variation. The efficiency of these elements is examined by comparing the results obtained utilizing the MARC software package.

Energy expression for compressible material is a combination of both strain energy and external workforce. This formulation is modified in such a way that the volume of the body remains the same before and after loading. To bring this behavior in the body the volumetric strain constraint is imposed. Mathematically, volumetric strain is written as

$$\frac{dV}{V}=0\ . \tag{3}$$

Assuming V = rzθ, where r is the radius, z is the element height, and θ is the rotational angle.

From the theory of elasticity, the stress-strain relation is

$$\varepsilon_r+\varepsilon_z+\varepsilon_\theta=(\sigma_r+\sigma_z+\sigma_\theta)\frac{1-2\vartheta}{E} \tag{4}$$

We get $\varepsilon_r+\varepsilon_z+\varepsilon_\theta=\frac{P}{K}$

This can be rewritten

$$\varepsilon_V-\frac{P}{K}=0$$

Where

K is the bulk modulus and ε_V is the volumetric strain and $P = (\sigma_r + \sigma_z + \sigma_\theta)/3$

Writing the total potential energy with imposing ε_V=P/K as a constraint will make the volume of the body remain constant before and after loading as K tends to infinity.

The modified total potential for incompressible materials is given by

$$\pi(\mathrm{u,P}) = 2\pi\left[\frac{1}{2}\int_A \{\varepsilon\}^T\{\sigma\}\mathrm{rdA} + \frac{P}{2}\int_A\left(\varepsilon_V - \frac{P}{K}\right)\mathrm{rdA} - \int_A \{u\}^T\{b\}\mathrm{rdA} - \int_S \{u\}^T\{t\}\mathrm{rdS}\right] - \sum_i u_i f_i \quad (5)$$

Strain and Stress vectors for the axisymmetric body are given by

$\{\varepsilon\} = \{\varepsilon_d\} + \{1/3\}\varepsilon_V$

$\{\sigma\} = \{\sigma_d\} + P$

Where $\{\varepsilon_d\}$is the deviatoric strain, and $\{\sigma_d\} = \{\sigma\} - P$

Substituting for $\{\varepsilon\}$ and $\{\sigma\}$ in (5), the potential energy equation is modified as

$$\pi(\mathrm{u,P}) = 2\pi\left[\frac{1}{2}\int_A \{\varepsilon_d\}^T\{\sigma_d\}\mathrm{rdA} - \frac{1}{2}\int_A \frac{P^2}{K}\mathrm{rdA} + \frac{1}{2}\int_A P\{\varepsilon_V - 3\alpha\Delta T\}\mathrm{rdA} - \int_A \{u\}^T\{b\}\mathrm{rdA} - \int_S \{u\}^T\{t\}\mathrm{rdS}\right] - \sum_i u_i f_i \quad (6)$$

By Substituting

$\{u\} = [N]\{q\}$

$\{\varepsilon_d\} = [B_d]\{q\}$

$\{\sigma_d\} = [C_d][B_d]\{q\}$

$P = \{H\}^T\{p\}$

And taking variation with respect to u and p results in the following matrices.

$$\begin{bmatrix} [K_{uu}] & [K_{up}] \\ [K_{pu}] & [K_{pp}] \end{bmatrix} \begin{Bmatrix} \{q\} \\ \{p\} \end{Bmatrix} = \begin{Bmatrix} \{F_m\} \\ \{F_T\} \end{Bmatrix}$$

$$\{F_m\} = 2\pi\left[\int_A [N]^T\{b\}rdA - \int_1 [N]^T\{t\}ds\right]$$

$$\{F_T\} = 2\pi\int_A [H]^T\, 3\alpha\Delta T rdA$$

$$[K_{uu}] = 2\pi\int_A [B_d]^T[C_d][B_d]rdA$$

$$[K_{up}] = -2\pi\int_A [B_v]^T\{H_p\}rdA$$

$$[K_{pp}] = -2\pi\int_A \frac{1}{K}[H_p]^T\{H_p\}rdA$$

Where $\{q\}$ is the element displacement vector, $\{p\}$ is the elemental hydrostatic pressure, $\{F_m\}$ is the load vector due to mechanical load, $\{F_T\}$ is the load vector due to thermal load, α is the linear coefficient of thermal expansion, ΔT is

the temperature difference, $[K_{uu}]$ is the element displacement stiffness matrix, $[K_{pp}]$ element pressure stiffness matrix, $[K_{up}]$ is the cross coefficient stiffness matrix corresponding to $\{q\}$and $\{p\}$. If the displacement is assumed as quadratic variation, then the pressure is assumed as bilinear variation. In general, the pressure variation is assumed one order lower than the displacement variation.

The displacements are used to get the elemental strains, stresses at the Gauss point and then it is extrapolated to the nodal points. The Gauss point stresses and strains are extrapolated to the nodal points by the bilinear extrapolation matrices as given in [8]. The stresses and strains for 8-node and 9-node quadrilateral elements are computed at 2x2 gauss points and extrapolated to 4 corner nodes of the element [8]. Stresses and strains at rest of the nodes are obtained by averaging the corner nodal values.

VALIDATION

The developed elements are validated with a benchmark problem of a clamped cylinder subjected to thermal loading to study the accuracy of the code. Mesh convergence is studied by using a thick cylinder subjected to internal pressure. A thick cylinder is refined with finite element and error obtained with the analysis is plotted against mesh density.

$$E = \left[1 - \frac{\text{Actual}}{\text{Theory}}\right] X100 \tag{7}$$

Figure 1 and 2 shows the plots of % Error vs. Mesh density along the radial direction. As the mesh is refined along the radial direction the percentage error obtained converges to 0.

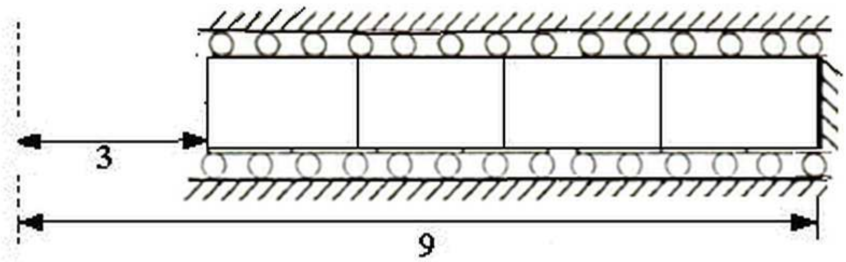

Figure 1: A Thick Cylinder Subjected to Thermal Loading

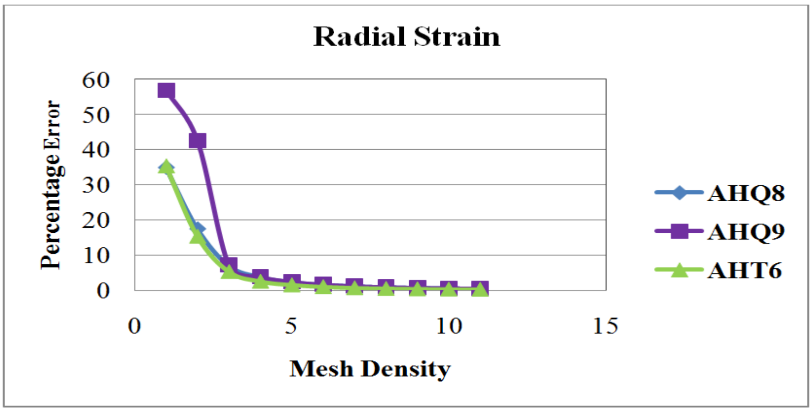

Figure 2: Plot of Percentage Error in Radial Strain vs Mesh Density

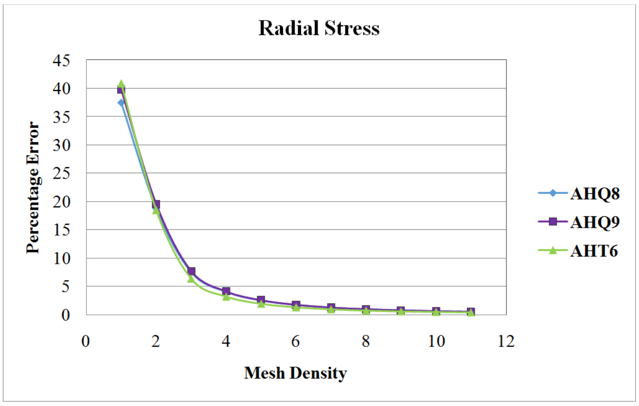

Figure 3: Plot Showing the Convergence of Radial Stress with an Increase in Mesh Density

Figure 1 shows the schematic configuration details with boundary conditions and finite element idealization. Each quadrilateral element is discretized into four AHT6 triangular elements. Material properties used are E = 1000, ν = 0.3, α = 0.0001 and the thermal load of ΔT = -38º C. Plots of radial displacement, radial strain, hoop strain, radial stress, and axial stress and hoop stress are obtained with present elements and compared with ANSYS plane 183 from inner port to outer port are shown in Figure 4 to 8 respectively.

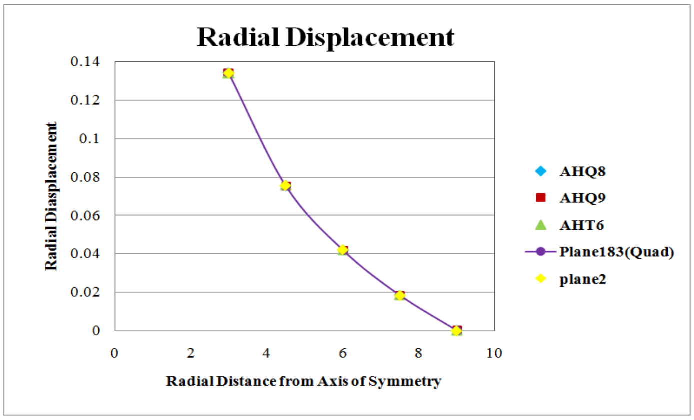

Figure 4: Plot Showing a Variation of Displacement from Inner Port to Outer Port

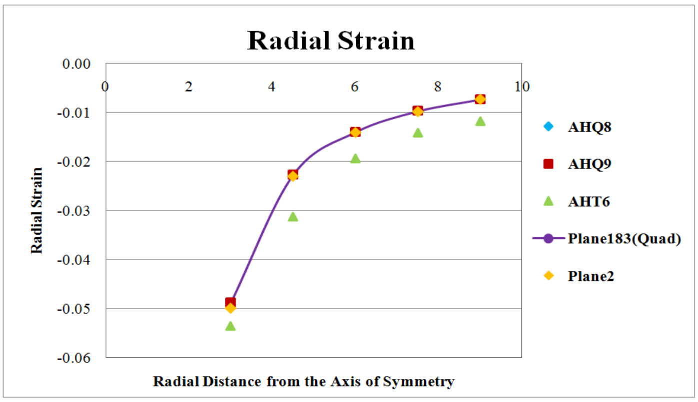

Figure 5: Plot Showing Variation of Radial Strain from Inner Port to Outer Port

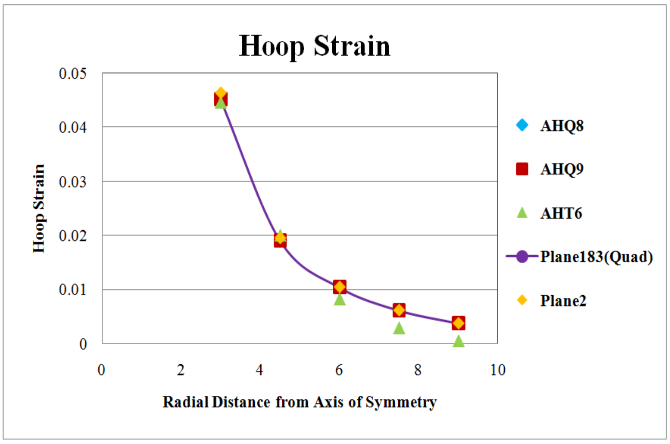

Figure 6: Plot Showing the Variation of Hoop Strain from Inner Port to Outer Port

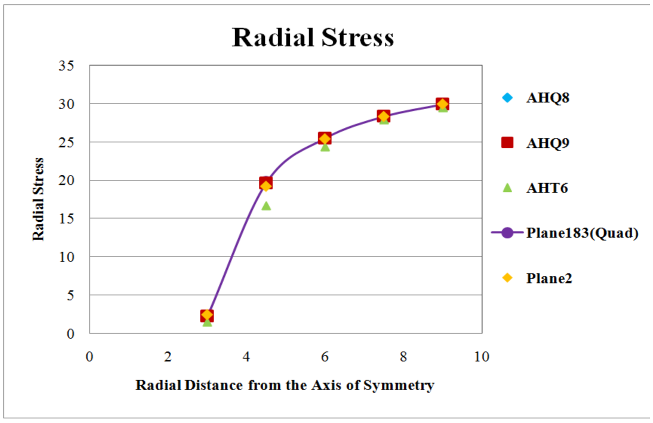

Figure 7: Plot Showing Variation of Radial Stress From Inner Port to Outer Port

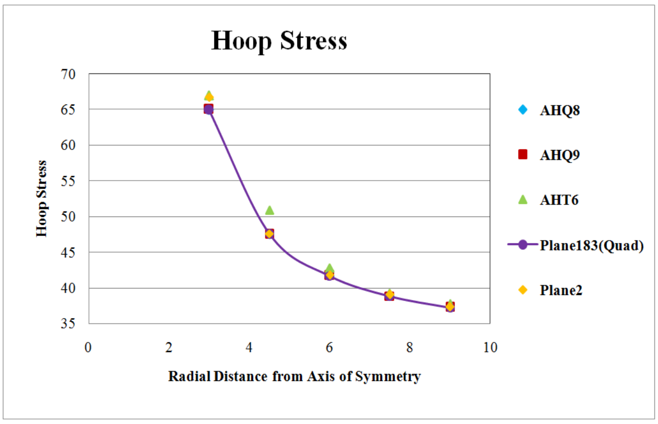

Figure 8: Plot Showing Variation of Hoop Stress from Inner Port to Outer Port

CASE STUDY: SOLID ROCKET MOTOR SUBJECTED TO THERMAL LOAD

The SRM is composed of viscoelastic solid propellant grain cast into a strong metallic or composite case and a liner in between the case and grain as an insulator. These motors are used mainly in defense and space applications and in the process, it is subjected to temperature change due to climatic variations of storage or the regions they are transported to.

The mechanical properties of the propellants are very sensitive to change in temperature, the thermal effects on the performance of the motor should be carefully examined before firing. Thermal and pressure loads during operation results in stress and strain which exceeds the material capabilities [9] [10] leading to the failure of the motor.

Thermal stresses are generated due to the cooling from the stress-free temperature to the storage temperature [9]. To simulate the stress and strain of the rocket motors (SRMs), an FEA model was established. Due to variations in the thermal expansion coefficients of the propellant and the casing, thermal stresses and strains are developed. Figure 9 shows the finite element model of a typical SRM with discretization and boundary conditions, which is analyzed for applied thermal load, T= -35°C using the elements developed using Herrmann Formulation. The material properties are given in Table 1.

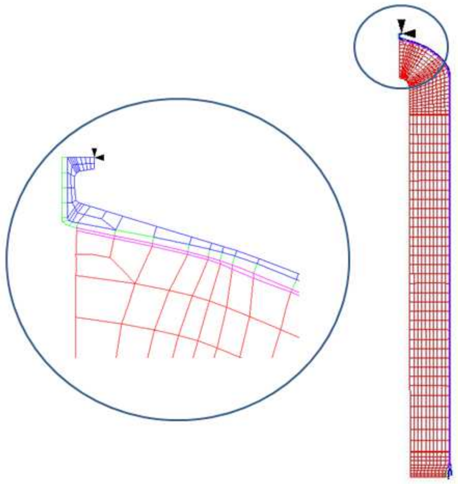

Figure 9: Finite Element Discretization and Boundary Condition for SRM

Table 1: Mechanical Properties of the Solid Propellant Rocket Motor Subjected to Thermal Cooling of -35°c

Material	Modulus (Kg/mm^2)	Poisson's Ratio	Coefficient of Thermal Expansion α (/°C)
Propellant	0.20	0.499	8.6E-5
Insulation	1	0.5	0.0001
Casing	2.08E4	0.3	1E-5

The results of the analysis are shown in the deformed and undeformed shapes (Figure 10) and the contour plots (Figure 11). The variation of resultant displacement, hoop strain at the inner port of propellant and the shear stress at the outer port is presented in path plots (Figure 12). The results show higher displacements at the head section of the rocket motor and at the lower end of the mid-section due to thermal shrinkage, which results in higher shear stresses. It is observed from the results obtained based on Herrmann formulation finite elements (present study) are in close agreement with those results obtained from MARC software.

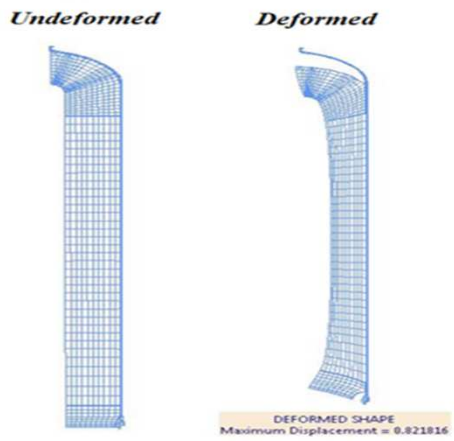

Figure 10: Undeformed Shape and Deformation Due to Thermal Shrinkage

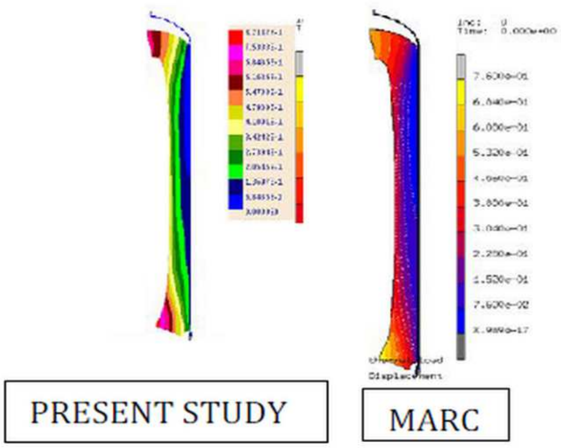

Figure 11: Resultant Deformation Contour Comparison

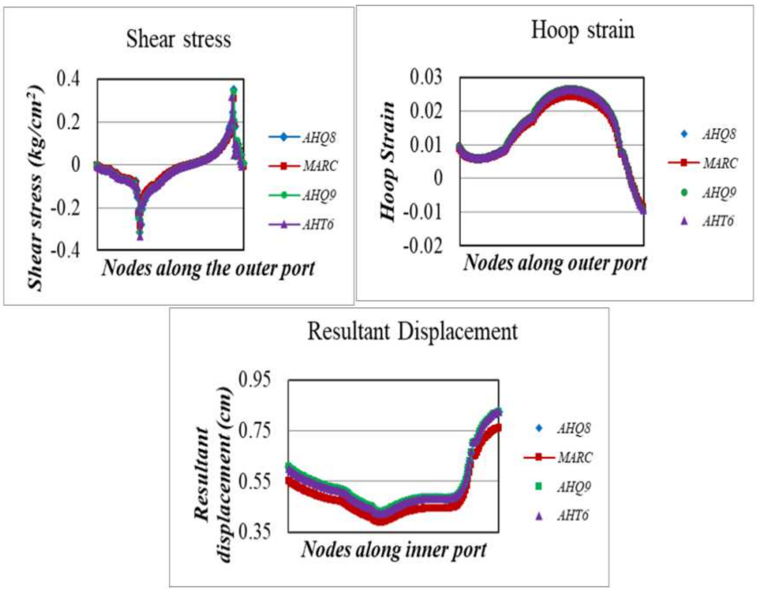

Figure 12: Variation of Shear Stress, Hoop Strain, and Displacement

CONCLUSIONS

Herrmann formulation based Axisymmetric FE on is validated by comparing the results with the eight-node quadrilateral axisymmetric Hermann element of MARC. It can be observed that while using MARC software package, the casing material (which is compressible in nature) is idealized using the standard eight-node is oparametric quadrilateral axisymmetric element having two degrees of freedom (w, u), while the propellant grain (which is nearly incompressible in nature) is idealized using the Hermann element having three degrees of freedom (w,u, σ_{mean}). Here σ_{mean} is the hydrostatic pressure known as the mean pressure. For the present case bonded cylindrical solid propellant grain, the interface nodes should be connected using tying option to take care of the mismatch between two degrees of freedom of casing element and three degrees of freedom of nearly incompressible propellant element [9]. The present study does not require tying of the nodes, Whether the structure is made of compressible or nearly incompressible materials. It can be concluded from the above-considered numerical problems that the present axisymmetric element can be used for examining the structural behavior of rocket motors having nearly incompressible and incompressible materials.

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